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Proof of a 'weak' form of Goldbach's Conjecture

The strong form of Goldbach's conjecture can be stated as: *Any even integer greater than 2 can be formed from the sum of two prime numbers*.

The 'weak' form of Goldbach's conjecture which I will prove below is: *There are at least* 2N-1 *unique even integers formed by the sum of pairs of odd primes using only the first* N *odd primes.*

Corollary: There are an infinite number of *unique* even integers which can be formed by the sum of two primes.

The strong form of Goldbach's conjecture remains unproven since the guaranteed unique infinite series of integers has gaps in it.

PROOF

Number the odd primes in ascending order {P1,P2,P3 … PN}, where P1=3, P2=5, P3=7, and so forth. Write all possible pairs in a table with the smallest prime on the left of the pair. Start at the top left and write a column of sums of P1 in ascending order. Then move to the next column and write sums of P2, and so on.

The resulting table is shown at the end of this paper. Note that the combinations are unique, but the resulting even integers are not all unique. By the nature of the labelling of the odd primes it is guaranteed that P1<P2<P3<P4 and so forth. Thus we can start at the top left corner (P1+P1) and move down the column producing a series of unique increasing even integers. It is obvious, for example that (P1+P3) > (P1+P2). Likewise we can move right across the table from any point and get an increase. For example (P3+P8) > (P2+P8).

Thus we get a series of unique increasing integers by moving through the table, starting at the top left corner and at every step moving either down or right. The longest such sequence is clearly 2N-1 elements long, and there are a multiplicity of such sequences.

If we consider the case for N=4, the straight "outside" path is:
P1+P1, P1+P2, P1+P3, P1+P4, P2+P4, P3+P4, P4+P4.

Another path is:
P1+P1, P1+P2, P2+P2, P2+P3, P3+P3, P3+P4, P4+P4.

… and there are another two equally valid monotonically increasing sequences of unique even integers. It is clear from the diagram that there are 2N-1 terms in each of these optimal length sequences. The proposed ‘weak’ form of Goldbach’s conjecture is therefore proved.

Discussion

It was convenient to ignore the sole even prime, 2, as it just destroys the symmetry of the argument. Clearly the only use for 2 in the Goldbach conjecture is to create 4 as 2+2. It has no further use and it is therefore best ignored.

It has been proved that there are 2N-1 unique even integers formed from the sum of two odd primes up to and including PN. Unfortunately PN grows (asymptotically) according to an $N \cdot \log_e(N)$ law, whereas the unique even integer sequence only grows as N. It is therefore evident that as N increases, the number of ‘holes’ in the unique even integer sequence will increase.

The total number of elements in the table is actually $\frac{N(N+1)}{2}$, so the number of elements in the table grows faster than the requirements for even integers in the range. This proves that as N gets larger there are an increasing number of duplicated even integers in the table, although it still doesn’t prove that all of the integers with the range are included.

Consider the relationship between the (down & left) diagonal elements in the table. For example we have diagonal groups such as: P4+P4, P3+P5, P2+P6, P1+P7. No rule has been presented to show if these sums are equal, increasing or decreasing along the diagonal. In fact looking at the numeric data, the evidence is that moving along any (down & left) diagonal the values can be equal, greater or lesser, with no obvious pattern. Again looking at the numeric data, there is no simple path through the table which covers all the even integers up to say PN+1.

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Table of Sums of Pairs of Odd Primes

P1+P1								
P1+P2	P2+P2							
P1+P3	P2+P3	P3+P3						
P1+P4	P2+P4	P3+P4	P4+P4					
P1+P5	P2+P5	P3+P5	P4+P5	P5+P5				
P1+P6	P2+P6	P3+P6	P4+P6	P5+P6	P6+P6			
P1+P7	P2+P7	P3+P7	P4+P7	P5+P7	P6+P7	P7+P7		
P1+P8	P2+P8	P3+P8	P4+P8	P5+P8	P6+P8	P7+P8	P8+P8	
P1+P9	P2+P9	P3+P9	P4+P9	P5+P9	P6+P9	P7+P9	P8+P9	P9+P9